Sec-B

Effect of Boundaries on Ground water :-

Boundary conditions are necessary to define the site specific ground water interacts with entire flow system. Boundaries are largely responsible for how flow occurs in the system.

* Physical boundaries causes changes in the faults, facies changes and surface water bodies.
* In Hydraulic boundaries, Ground water divides –
  At recharge or discharge areas
  Topographically high or low areas.

Specified Head Boundaries –
Hydraulic head is given for the boundary.

Specified flow boundaries –
* Flux across the boundary is given.
* A no-flow boundary has a flux of zero.

Interference of water
OR
Interference Among Wells (confined Aquifer)

If two or more wells are constructed in such a way that they are near to each other for discharging and their

P.T.O.

drawdown curves intersect within their radius of zero drawdown, they are said to interfere. Such interference of wells decreases the discharge of such interfering wells.

<u>Leaky Aquifers</u> :-

Aquifers, whether artesian or water table, that lose or gain water through adjacent less permeable layers is known as Leaky Aquifers.

Leaky Aquifer or semi-confined aquifer is similar to a confined aquifer, except that the aquitards next to it are more permeable and let significant amounts of water move through to the aquifer.

Thiem's Equilibrium Formula for unconfined Aquifer.

Let a non-Artesian (unconfined) well be driven, and water pumped heavily so as to cause sufficient ~~cone~~ drawdown.

~~when the water~~

Let the two observation wells lying with in the circle of influence of the main pumped well are to be driven.

Let these wells be numbered as 1 and 2 as shown in fig. and let them be at distances of r_1 and r_2 from the main well. Let d be the depth of the well or the aquifer below the static water table.

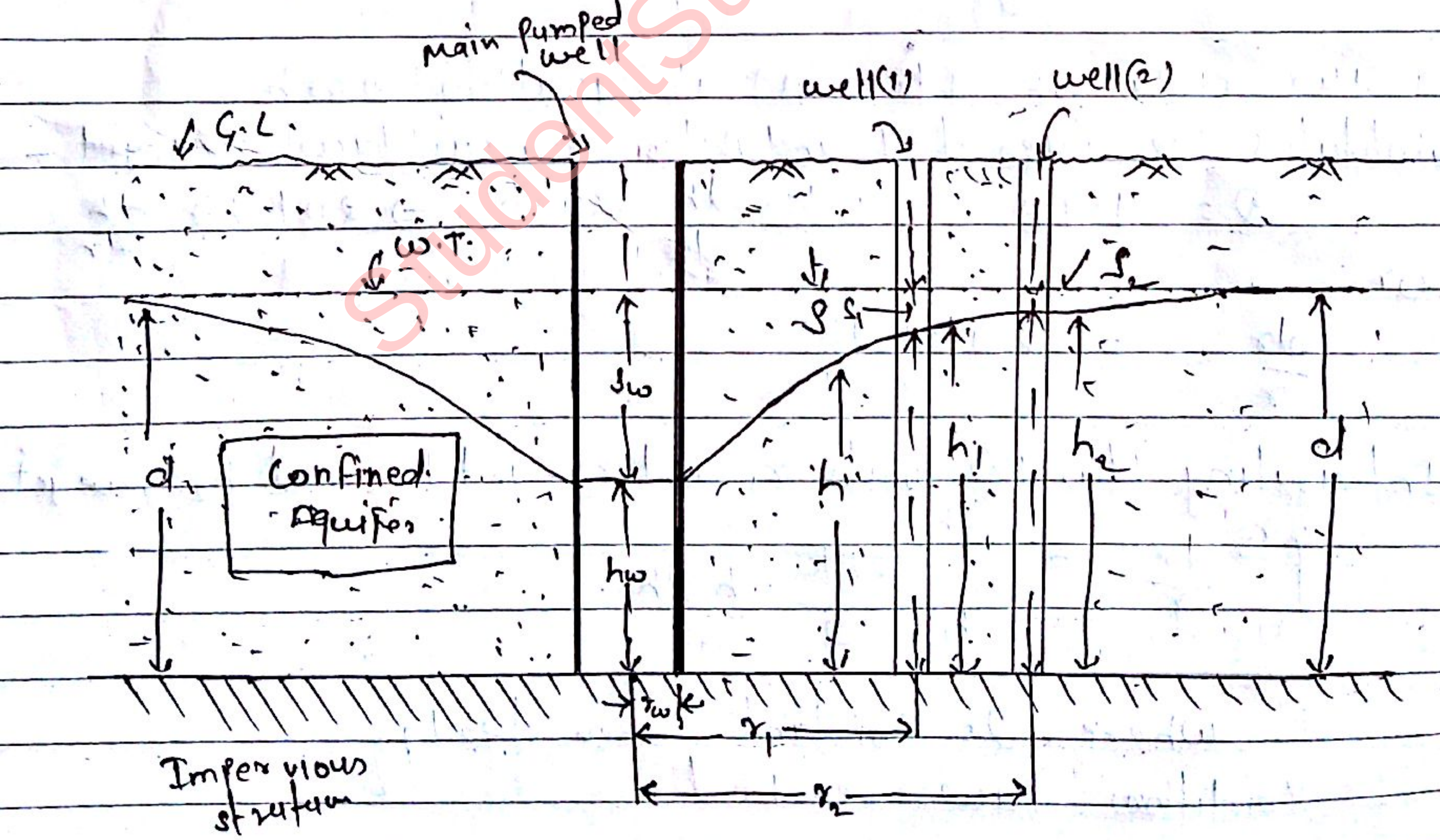


Fig - Unconfined Aquifer Case of ~~Thiem's~~ eq^n -

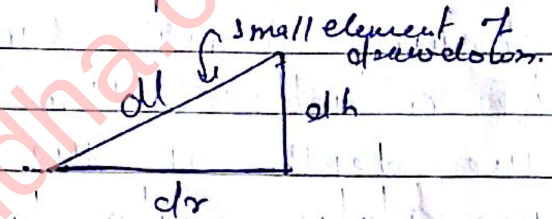
Let s_1 and s_2 be the drawdowns in the two corresponding observation wells

From Darcy's law –

$$Q = KIA \quad \text{--- (1)}$$

where I = Hydraulic gradient
using cylindrical co-ordinates, we take r as the radius of cylinder and h as the height of the cone of depression at a distance r from the main well.

$$I = \frac{dh}{dl} = \frac{dh}{dr}$$



$$I = \frac{dh}{dr}$$

the area of flow (A) is equal to $2\pi r h$
substituting the values of I and A in Darcy's law, we get –

$$Q = KIA = K\frac{dh}{dr}2\pi r h \Rightarrow 2\pi K h r \frac{dh}{dr}$$

or

$$\frac{dr}{r} = \frac{2\pi K h\,dh}{Q}$$

Integrating b/w the limits r_1 and r_2 and h_1 and h_2, we get,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2\pi K}{Q} \cdot h\,dh$$

where Q = constant when steady conditions have reached, Therefore, $\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{Q} \int_{h_1}^{h_2} h\,dh$

or $\left|\log_e r\right|_{r_1}^{r_2} = \frac{2\pi k}{Q} \left|\frac{h^2}{2}\right|_{h_1}^{h_2}$

$\Rightarrow \log_e \frac{r_2}{r_1} = \frac{2\pi k}{Q}\left[\frac{(h_2^2 - h_1^2)}{2}\right] = \frac{\pi k}{Q}\left[h_2^2 - h_1^2\right]$

or $k = \frac{Q \cdot \log \frac{r_2}{r_1}}{\pi (h_2^2 - h_1^2)}$

$$\boxed{Q = \frac{\pi k (h_2^2 - h_1^2)}{2.3 \log_{10} \frac{r_2}{r_1}}} \quad \text{—} \quad (2)$$

But $(h_2^2 - h_1^2) = (h_2 + h_1)(h_2 - h_1)$ and $h_2 - h_1 = s_1 - s_2$

Also

$h_1 + h_2 = d + d = 2d$

or $(h_2^2 - h_1^2) = 2d(s_1 - s_2)$

Putting this value in eq^n(2), we get

$Q = \frac{2\pi k \cdot d (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}}$

$[kd = T]$

$$\boxed{Q = \frac{2\pi T (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}}} \quad \text{—} \quad (3)$$

Thiem's Equilibrium formula for Confined Aquifer :-

~~The~~ Formula used for the case of the unconfined aquifer has to be slightly modified in the case of confined aquifer.
In a confined aquifer, the flow is actually radial and horizontal and therefore, it has not to be assumed as such, as it was in the unconfined ~~to~~ aquifer case.

From Darcy's law —

$$Q = KIA = K \cdot \frac{dh}{dr} \, 2\pi r H$$

H = Height of confined aquifer

or $$Q = 2\pi K H r \cdot \frac{dh}{dr}$$

or

$$\frac{dr}{r} = \frac{2\pi K H}{Q} \cdot dh$$

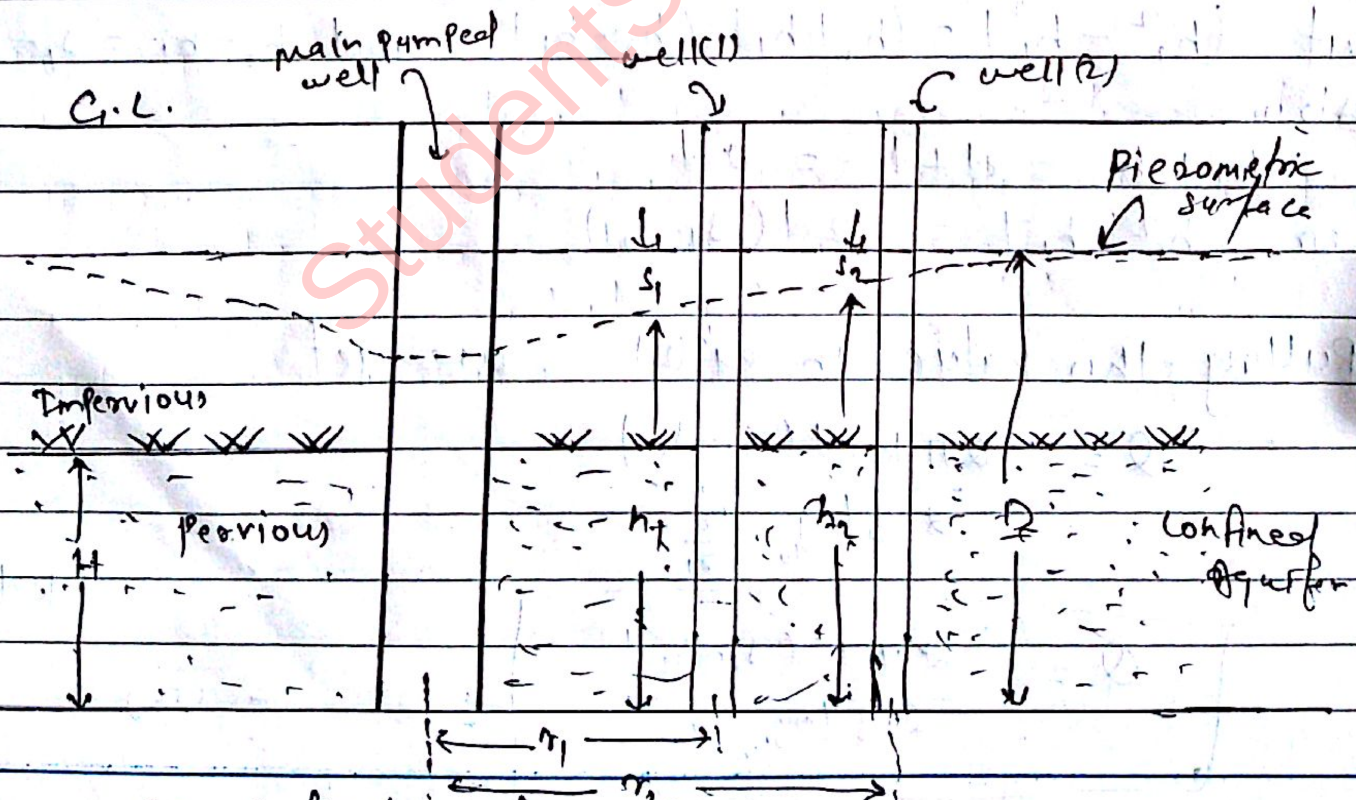


fig confined aquifer case ~~of~~ for thiem's eqn —

Integrating b/w the limits of r_1 and r_2, we get

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k H}{Q} \int_{h_1}^{h_2} dh$$

or

$$\left| \log_e r \right|_{r_1}^{r_2} = \frac{2\pi k H}{Q} \left| h \right|_{h_1}^{h_2}$$

or

$$\log_e \frac{r_2}{r_1} = \frac{2\pi k H}{Q} \left[h_2 - h_1 \right]$$

$$\boxed{Q = \frac{2\pi k H (h_2 - h_1)}{2.3 \log_{10} \frac{r_2}{r_1}}}$$

But $h_2 - h_1 = s_1 - s_2$

$\therefore$

$$Q = \frac{2\pi k H (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}}$$

Also

$$\boxed{Q = \frac{2\pi T (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}}}$$

$[KH = T]$

Limitations of Thiem's Equilibrium formula's :-

In actual practice -

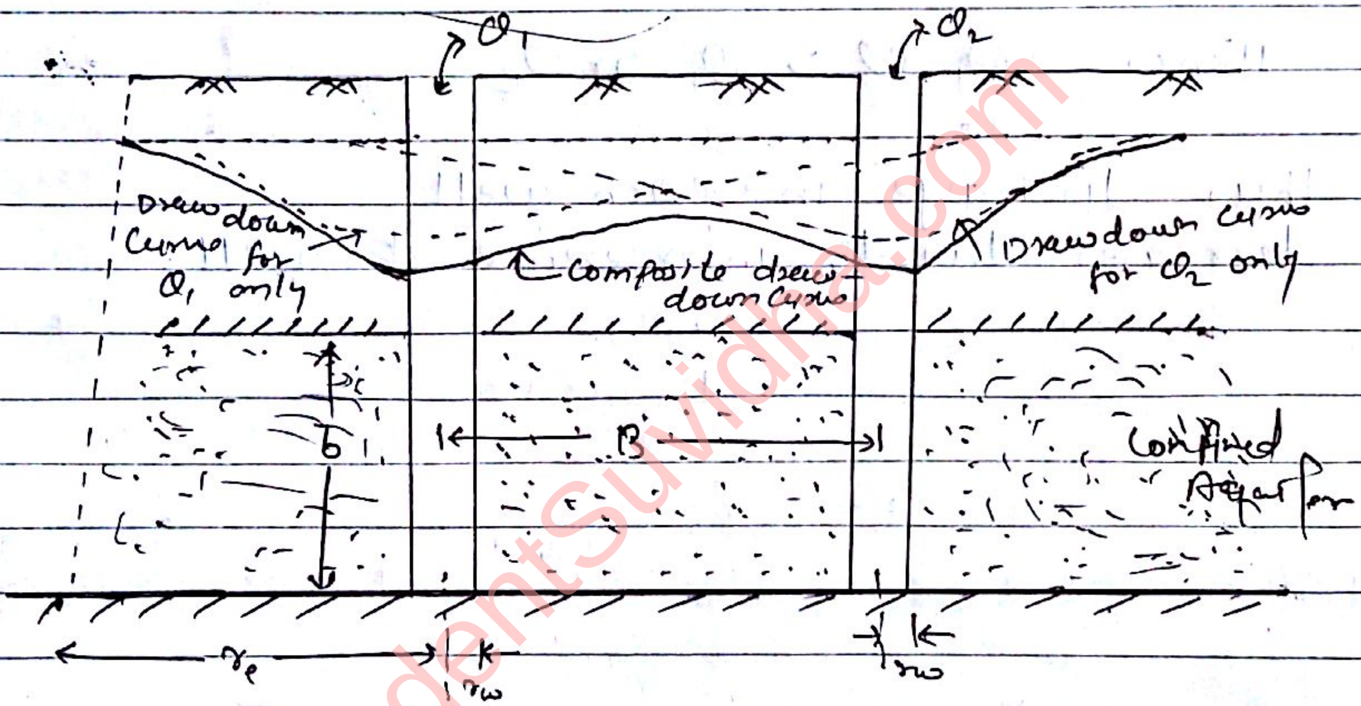
* An aquifer is not fully homogeneous
* The well might have been dug half away through the aquifer.
* permeability may not be uniform
* The ground water table may be inclined and thus, the base of the cone may not be a circle
* The equilibrium conditions might have not fully reached.

Assumptions :-

* The aquifer is homogeneous, isotopic and of infinite and areal extent
* The well has been sunk through the full depth of the aquifer
* pumping has continued for a sufficient time at a uniform rate, so that the equilibrium stage or steady flow conditions have reached.
* Flowlines are radial and horizontal and flow is laminar.

Interference of wells :-

When two wells situated near to each other are discharging their drawdown curves intersect within their radius of zero drawdown.
Fig shows interference b/w two wells



The two wells are at a distance B apart having same diameter, drawdown and discharge over the same period of time.
Let the depth of the aquifer is b.
By using the method of complex variable, the discharge in each well is given by -

$$Q_1 = Q_2 = \frac{2\pi k b (r_e - r_w)}{\log_e \frac{r_e^2}{r_w B}}$$

where r_e is the radius of influence

$r_e >>> B$

Since

$r_e >>> B$, $\therefore \frac{r_e^2}{r_w B} > \frac{r_e}{r_w}$

Hence $\cancel{Q_1}$ $Q' > Q_1$ or Q_2

Thus discharge in each well decreases due to interference of well.

Partial Penetration of an Aquifer by a well:-

In a well when the intake of the well is less than the thickness of the well, then the well is called partially penetrated well.
In case of partially penetrated well, the flow lines are not truly horizontal near the well. The flow lines are curved upward or downward near the well.
The drawdown in case of partially penetrating well is more than the fully penetrating well.
Fig. shows a partially penetrated well.

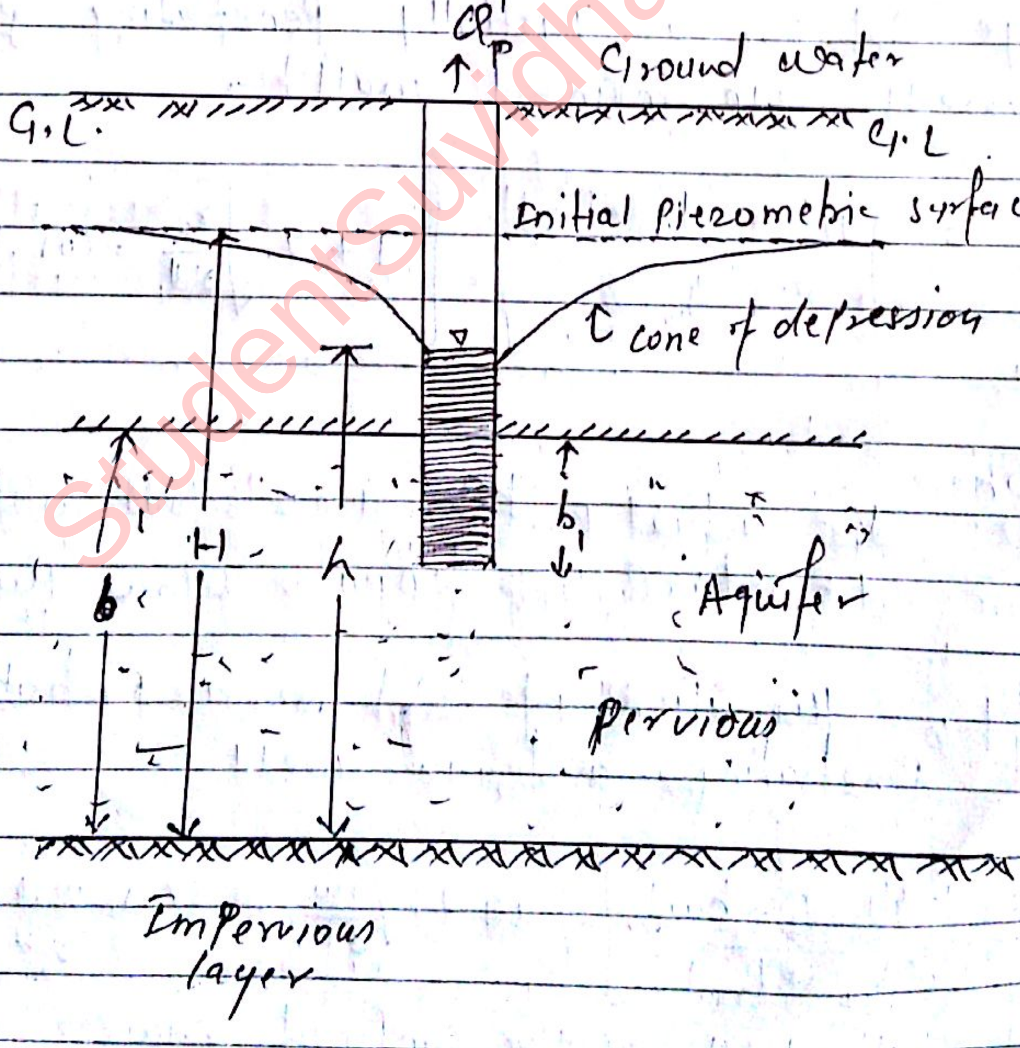


Fig - Partial Penetration of An Aquifer

From Dupuit's formula —
we have

$$Q = \frac{\pi k (d^2 - h_\omega^2)}{2.3 \log_{10}\left(\frac{R}{r_\omega}\right)} \quad \text{(for unconfined wells)}$$

$$Q = \frac{2\pi k H (D - h_\omega)}{2.3 \log_{10}\left(\frac{R}{r_\omega}\right)} \quad \text{(for confined well)}$$

Thus,
Discharge (Q_p) for a partially penetrating gravity well (unconfined well) —

$$= \left[\frac{\pi k \cdot (d_1^2 - h_\omega^2)}{2.3 \log_{10}\left(\frac{R}{r_\omega}\right)}\right]\left[1 + 7 \cdot \sqrt{\frac{r_\omega}{2d_1}} \cdot \cos\frac{\pi d_1}{2d}\right] \times \frac{d_1}{d}$$

where
d_1 = actual penetration depth below water table
d = Actual depth of Aquifer below the W.T.

Similarly, the discharge (Q_p) for a partially penetrating artesian well

$$= \left[\frac{2\pi k H_1 (D - h_\omega)}{2.3 \log_{10} (R / r_\omega)}\right]\left[1 + 7 \cdot \sqrt{\frac{r_\omega}{2H_1}} \cos\frac{\pi H_1}{2H}\right] \times \frac{H_1}{H}$$

H = full depth of the confined Aquifer
H_1 = Depth upto which the well penetrates

Change in hydraulic properties near a well :-

Consider a case of pumping well as shown in fig -

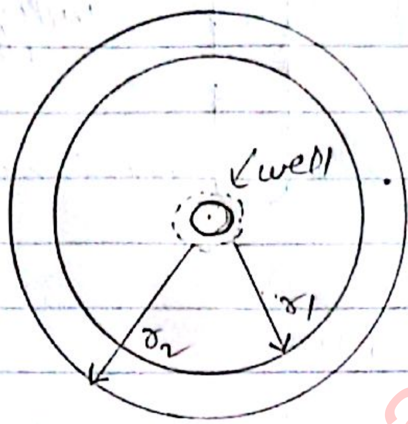


fig. Pumping well.

The discharge of the well can be expressed as -

$$Q = A_1V_1 = A_2V_2$$
$$= 2\pi r_1 h V_1 = 2\pi r_2 h V_2$$
$$\Rightarrow r_1 V_1 = r_2 V_2$$

Here $r_2 > r_1$

so, $V_1 > V_2$

Therefore, velocity near the well is more than the velocity away from the well.

Due to the high velocity in the vicinity of the well, the fine particles that are present in the aquifer formation are moved with the flow of water.

Spherical flow in a well :-

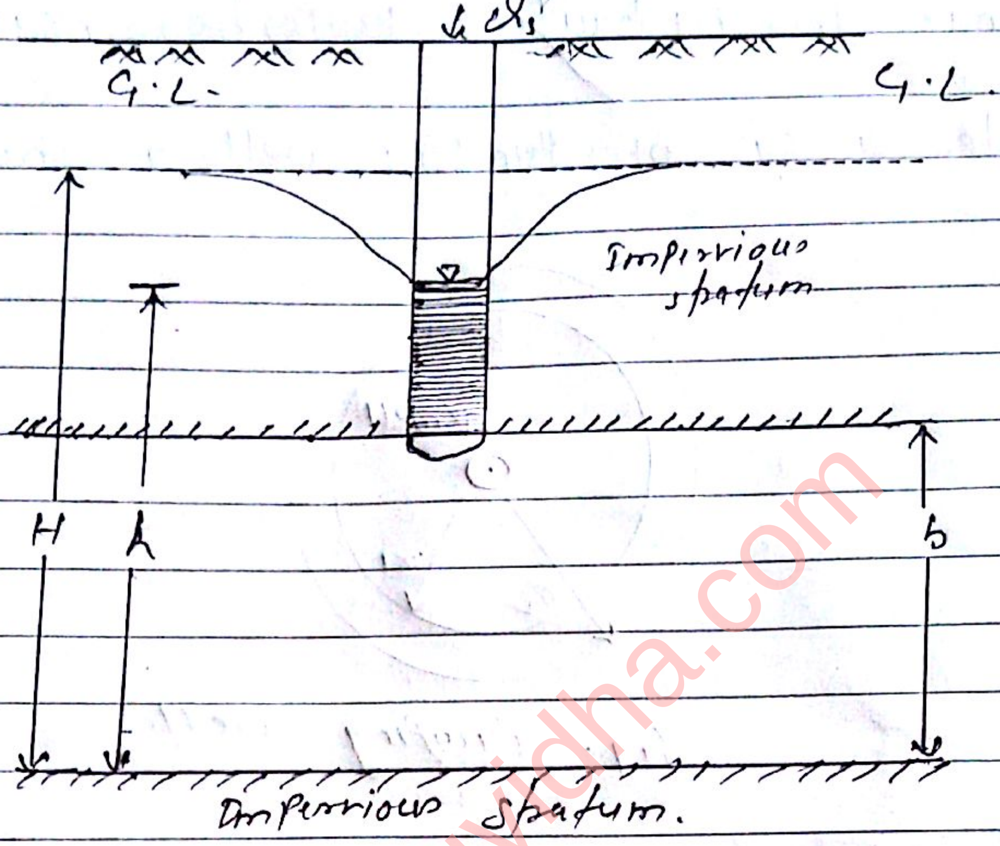


Fig-

Fig shows a special case of partially penetrating well, where the well just penetrates up to the top surface of the semi- infinite porous medium.

In this case, the flow towards the well becomes purely spherical.

The discharge Q_s from such a well can however be calculated from the eq^n —

$$Q_s = 2\pi k r_w (D - h_w) \quad \text{—— (11)}$$

whereas in the case of a simple radial flow in a fully penetrating well, the discharge is given by the eq^n ——

$$Q = \frac{2\pi K \cdot H (D - h_w)}{2.3 \log_{10}\left(\frac{R}{r_w}\right)}$$

Now

$$\frac{Q_s}{Q} = \frac{2\pi k r_w (D - h_w)}{\left(\frac{2\pi k H (D - h_w)}{2.3 \log_{10} \frac{R}{r_w}}\right)} = \frac{2.3 r_w \log_{10}\left(\frac{R}{r_w}\right)}{H}$$

or

$$\boxed{\frac{Q_s}{Q} = 2.3\left(\frac{r_w}{H}\right) \log_{10}\left(\frac{R}{r_w}\right)} \quad \text{— } (2)$$

$$2.3 \log_{10} (R/0.3)$$

or $$1.36 = \frac{1.603 \times 10^{-3} \times 157.4 \times 22.6}{2.3 \log_{10} (R/0.3)}$$

or $$\log_{10} (R/0.3) = \frac{1.603 \times 157.4 \times 22.6}{2.3 \times 1{,}360} = 1.824$$

Taking antilog, we get

$$\frac{R}{0.3} = 66.7 \quad \text{or} \quad R = 20.01 \quad \text{Say} \quad \mathbf{R = 20\ m.}$$

Now, specific capacity = $Q_{\text{unit drawdown}} = \dfrac{\pi K\,[(90)^2 - (89)^2]}{2.3 \log_{10} (20/0.3)}$

$$= \frac{1.603 \times 10^{-3} \times 179 \times 1}{2.3 \times 1.824} = 68.3 \times 10^{-3} \text{ m}^3/\text{min}.$$

Hence, the specific capacity = **68.3 litre/minute. Ans.**

(*d*) *Maximum discharge* will occur when

$$h_w = 0$$

$$\therefore \quad Q_{max} = \frac{\pi K\,[(90)^2 - (0)^2]}{2.3 \log_{10} (20/0.3)}$$

$$= \frac{1.603 \times 10^{-3} \times 8100}{2.3 \times 1.824} = 3.09 \text{ m}^3/\text{min}$$

Hence, the maximum rate of discharge = **3,090 litres/minute. Ans.**

16.10. Non-Equilibrium Formula for Aquifers (Unsteady Radial Flows)

The main drawback of the equilibrium formulas given by Thiem and Dupuit, was the problem to attain equilibrium conditions, since it is not an easy job to do so. The pumping has to be continued at a uniform rate for a very long time so as to achieve steady flow conditions. Further research was, therefore, carried out to simplify the process and to calculate the yield in some other way.

A major advancement in this field was made by Thies when he developed his non-equilibrium formula by introducing the time factor t. This formula was derived by Thies in 1935, by comparing the flow of water with the flow of heat by conduction. The formula evolved by him involved the solution of a complicated integral, the solution of which requires the use of various tables and graphs.

Later, Jacob derived the same formula by directly using the hydraulic concept. He also slightly modified the Thies formula by making a slight approximation, so as to simplify it. The final formula which was arrived at the end, by Jacob, is given as :

$$s = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt}{r^2 . A} - 0.5772\right]^{*} \quad \text{...(16.46)}$$

where s = Drawdown in the observation well after a time t, from the start of pumping in the main well

T = Coefficient of transmissibility of the aquifer

Q = Constant discharge pumped out from the main pumped well.

A = Coefficient of storage of the measured drawdown.

r = Radial distance of the observation well from the main pumped well.

If, in an observation well at a distance r from the main pumped well, the drawdowns are respectively s_1 and s_2 at times t_1 and t_2 after the pumping was started in the main well, then

$$s_1 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_1}{r^2 . A} - 0.5772\right] \quad \text{...(i)}$$

and

$$s_2 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_2}{r^2 . A} - 0.5772\right] \quad \text{...(ii)}$$

Subtracting (*i*) from (*ii*), we get

$$s_2 - s_1 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_2}{r^2 A} - \log_e \frac{4Tt_1}{r^2 A}\right] = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_2}{r^2 A} \cdot \frac{r^2 A}{4Tt_1}\right]$$

$$= \frac{Q}{4\pi T}\log_e \frac{t_2}{t_1} = \frac{2.3\,Q}{4\pi T}\log_{10}\frac{t_2}{t_1}$$

or

$$s_2 - s_1 = \frac{2.3\,Q}{4\pi T}\log_{10}\frac{t_2}{t_1} \quad \text{...(16.47)}$$

The above formula holds good for larger values of t. It is evident from the above equation that if the drawdowns are noted for various values of t on the given observation well at a distance r from the main pumped well, and a graph is plotted between $\log t$ and s, it will be a straight line, with the limitation that the initial values (*i.e.* when t is small) may not exactly lie on a straight line. From this straight line, two values, of t_1 and t_2 and corresponding values of s_1 and s_2 can be read out, and knowing the value of T, Q can be worked out easily by using Eq. (16.44).

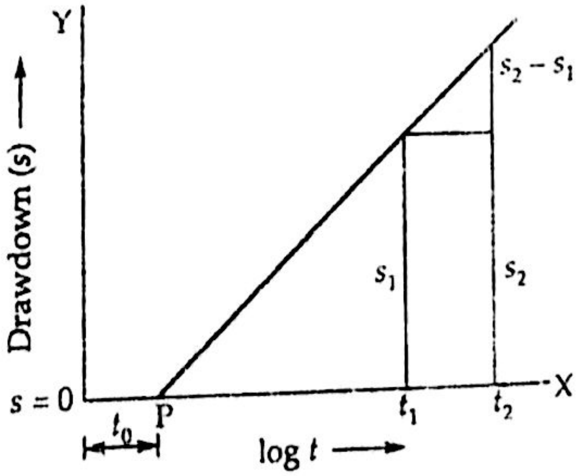


Fig. 16.24

*The derivation of the above formula is available in *"Water Resources Engg. Vol I—Hydrology and Water Resources Engg"*, and may be referred to in specific needs.

The straight line can be produced back, so as to cut the X-axis at point P, as shown in Fig. 16.24. Now let the value of t at P be represented by t_0.

Now, we know that

$$s = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt}{r^2 . A} - 0.5772\right]$$

at $\quad s = 0, t = t_0$

$$\therefore \quad 0 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_0}{r^2 A} - 0.5772\right]$$

Since $\quad \frac{Q}{4\pi T} \neq 0$

$$\log_e \frac{4Tt_0}{r^2 A} - 0.5772 = 0 \quad \text{or} \quad \log_e \frac{4Tt_0}{r^2 A} = 0.5772$$

or $$\frac{4Tt_0}{r^2 A} = e^{0.5772} \quad \text{or} \quad A = \frac{4Tt_0}{r^2 e^{0.5772}} = \frac{2.25Tt_0}{r^2}$$

Hence, $$\boxed{A = \frac{2.25Tt_0}{r^2}} \qquad ...(16.48)$$

The coefficient of storage A, can be worked out by this equation.

Example 16.10. *In an artesian aquifer, the drawdown is 1.2 metres at a radial distance of 10 metres from a well after two hours of pumping. On the basis of Thies' non-equilibrium equation, determine the pumping time for the same drawdown (i.e: 1.2 m) at a radial distance of 30 metres from the well.*

Solution. The Thies' non-equilibrium equation (16.43) is

$$s = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt}{r^2 . A} - 0.5772\right]$$

In the given question, the drawdown is the same in both the observation wells, therefore,

well (1),	well (2)
$r_1 = 10$ m,	$r_2 = 30$ m
$t_1 = 2$ hr.,	$t_2 = ?$

Now $$s_1 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_1}{r_1^2 . A} - 0.5772\right]$$

$$s_2 = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_2}{r_2^2 . A} - 0.5772\right]$$

But $\quad s_1 = s_2$

$$\therefore \quad \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_1}{r_1^2 A} - 0.5772\right] = \frac{Q}{4\pi T}\left[\log_e \frac{4Tt_2}{r_2^2 A} - 0.5772\right]$$

or $$\log_e \frac{4Tt_1}{r_1^2 A} = \log_e \frac{4Tt_2}{r_2^2 A}$$

or
$$\frac{4Tt_1}{r_1^2 A} = \frac{4Tt_2}{r_2^2 A}$$

or
$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} \qquad ...(16.49)$$

Putting the respective values, we get

or
$$\frac{2 \text{ hr}}{(10)^2} = \frac{t_2}{(30)^2} \quad \text{or} \quad t_2 = \frac{(30)^2}{(10)^2} \times 2 \text{ hr} = 9 \times 2 \text{ hr} = 18 \text{ hr}$$

t_2 **= 18 hr. Ans.**

Example 16.11. *A well is located in a 30 m thick confined aquifer of permeability 35 m / day and storage coefficient of 0.004. If the well is pumped at the rate of 1500 litres per minute, calculate the drawdown at a distance of 40 m from the well after 20 hours of pumping.* *(Civil Services, 1991)*

Solution. Using Jacob's eqn. (16.46), we have

$$s = \frac{Q}{4\pi T}\left[\log_e \frac{4T \cdot t}{r^2 A} - 0.5772\right]$$

where s = drawdown = ?

H = depth of aquifer = 30 m

K = 35 m/day

A = storage coeff = 0.004

$Q = 1500$ l/min $= \frac{1.5}{60}$ m³/sec = **0.025 m³/s**

r = 40 m

t = 20 hr = 20 × 3600 secs = 72000 secs

$T = K \cdot H$ = (35 × 30) m²/day

$= \frac{1050}{60 \times 60 \times 24}$ m²/sec = 0.012153 m²/s

Substituting values, we get

$$s = \frac{0.025}{4 \times 3.14 \times 0.012153}\left[\log_e \frac{4 \times 0.012153 \times 72000}{(40)^2 \times 0.004} - 0.5772\right]$$

$= 0.163\,[6.3042 - 0.5772] = 0.163 \times 5.724 = \mathbf{0.94}$ **m. Ans.**

WELLS

A water well is a hole usually vertical, excavated in the Earth for bringing ground water to the surface. The wells may be classified into two types :

(1) Open wells ; and (2) Tube wells.

16.20. Open Wells or Dug Wells

Smaller amount of ground water has been utilised from the ancient times by open wells. Open wells are generally open masonry wells, having comparatively bigger diameters, and are suitable for low discharges of the